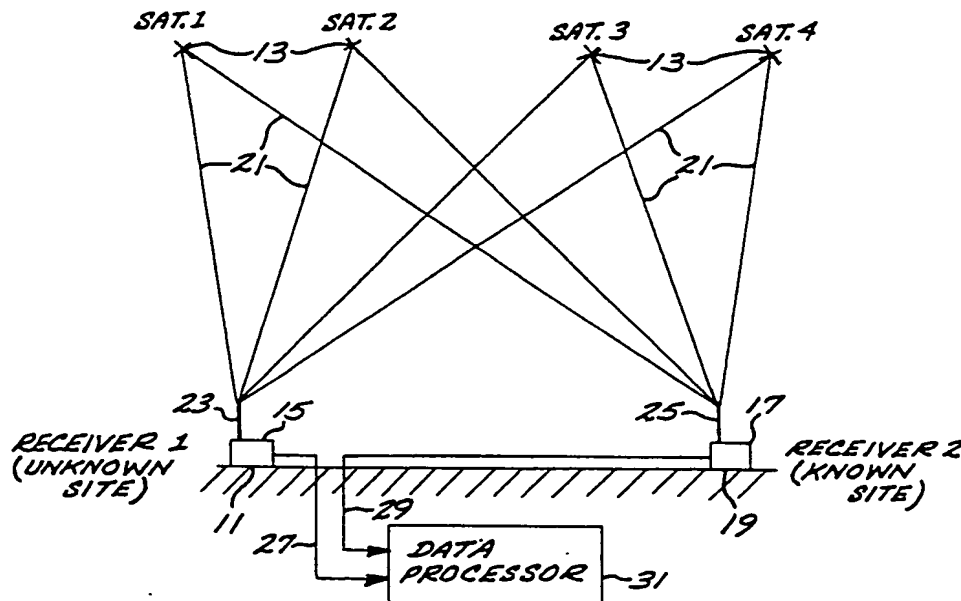




## INTERNATIONAL APPLICATION PUBLISHED UNDER THE PATENT COOPERATION TREATY (PCT)

<b>(51) International Patent Classification 4 :</b> <b>H04B 7/19</b>	<b>A1</b>	<b>(11) International Publication Number:</b> <b>WO 87/ 06410</b> <b>(43) International Publication Date:</b> 22 October 1987 (22.10.87)
<b>(21) International Application Number:</b> PCT/US87/00825 <b>(22) International Filing Date:</b> 15 April 1987 (15.04.87) <b>(31) Priority Application Number:</b> 852,988 <b>(32) Priority Date:</b> 15 April 1986 (15.04.86) <b>(33) Priority Country:</b> US  <b>(71) Applicant:</b> MAGNAVOX GOVERNMENT AND INDUSTRIAL ELECTRONICS COMPANY [US/US]; 1313 Production Road, Fort Wayne, IN 46808 (US).  <b>(72) Inventor:</b> HATCH, Ronald, R. ; 2829 Maricopa Street, Torrance, CA 90503 (US).  <b>(74) Agent:</b> PETTIT, George, R.; Pollock, Vande Sande & Priddy, 1990 M Street, N.W. #800, P.O. Box 19088, Washington, DC 20036 (US).		<b>(81) Designated States:</b> AU, BE (European patent), CH (European patent), DE (European patent), FR (European patent), GB (European patent), IT (European patent), JP, NL (European patent), SE (European patent).  <b>Published</b> <i>With international search report.</i>

**(54) Title:** METHOD AND APPARATUS FOR PRECISION SURVEYING USING BROADCAST SATELLITE SIGNALS



**(57) Abstract**

A method and apparatus for accurately and precisely determining the position coordinates of a receiver (15) located at a predetermined site using signals broadcast from a plurality of orbiting satellites (13) and using information derived from a second receiver (17) located at a reference site having known coordinates. Carrier phase measurements made by the two receivers on all of the incoming satellite signals are incorporated into a special square root information filter matrix (31) that permits a convenient determination of phase noise in the satellite and receiver clocks, for use in subsequently correcting the phase measurements. The position coordinates of the unknown site are determined in a special three-stage

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METHOD AND APPARATUS FOR PRECISION  
SURVEYING USING BROADCAST SATELLITE SIGNALS

Background of the Invention

This invention relates generally to surveying using signals broadcast from a plurality of orbiting satellites, and, more particularly, to satellite-based surveying systems that determine the position coordinates of an unknown site relative to a reference site whose position coordinates are known.

Satellite-based positioning systems such as the Global Positioning System (GPS) are now a highly popular means of accurately and precisely determining the position of a receiver. These systems have numerous practical applications and, depending on the time duration over which measurements are taken, they can determine a receiver's position to sub-centimeter accuracy.

In GPS, a number of satellites orbiting the earth in well defined polar orbits continuously broadcast signals indicating their precise orbital positions. The broadcast signals all have a common frequency, but are modulated by unique, pseudorandom digital codes. Each satellite signal is based on a precision internal clock. The receivers detect the superimposed modulated carrier signals and determine either or both of the code phase and carrier phase of each detected signal, relative to their own internal clocks. These detected phases can be used to determine the receivers' position coordinates.

One typical system for processing these carrier phase measurements is described in an article by Bossler et al, entitled "Using the Global Positioning System (GPS) for Geodetic Positioning", Bulletin Geodesique, Vol. 54, No. 2, 1980, pages 553-563. This article describes a processing technique known as double differencing. In this technique,

the carrier phase measurements are collected at each of two receivers for a plurality of GPS satellites. One receiver is located at a site whose position coordinates are to be determined, and the other is located at a reference site  
5 whose position coordinates are known. In an initial step, the carrier phase measurements for each satellite are differenced across the two sites. This eliminates by cancellation any clock error in the satellite, since the effects of such an error would be identical at each site.  
10 Thereafter, one satellite is chosen as a reference satellite, and the single difference measurement obtained for it is subtracted from the single difference measurements obtained for all of the other satellites. This second differencing eliminates by cancellation any clock errors in  
15 the receivers at the two sites, since the effects of such clock errors would be identical in all of the single differences measurement.

Several additional steps involving the double difference measurements are required. First, rough estimates for the  
20 unknown site's coordinates are provided to the apparatus. These estimated coordinates are compared with the specific satellite orbits obtained from the data broadcast by the satellite or from any other suitable source (e.g., the National Geodetic Survey). The expected value of the double  
25 difference measurements can thereby be determined. Equations defining the error in the expected double difference values are then formed by differencing them from the corresponding actual double difference measurement. The sensitivity of these error equations to changes in the  
30 estimated coordinates is also determined, whereby the error equations can be solved. The equations are solved usually in an iterative, least mean square error procedure, to determine the particular site coordinates that will minimize the total mean square error in the computed double difference values.

The carrier phase measurements do not reflect the initial number of integral carrier cycles present in each satellite/receiver link. The double difference measurements, therefore, are biased by an unknown integral number of cycles. The iterative, least squares procedure described above must, therefore, also solve for this bias in each double difference measurement.

When the iterative procedure described above has been iterated to the point where the position coordinates and double difference bias values have converged to fixed values, the procedure advances to an additional stage in which the computed double differences values are adjusted by the nearest whole value to the bias solution and then that bias state is dropped from the least squares computation. Typically, the uncertainty in the bias determination is also computed and it is dropped from the least squares computation only when the uncertainty is acceptably small. Otherwise, the iteration is continued.

The most accurate determination of the unknown site's position coordinates is obtained when an account is made for the correlation between the double difference measurements that arises from differencing with respect to the same satellite/receiver link. Accounting for this correlation significantly increases both the complexity and time required to complete the computation. In addition, these double differencing techniques are not ordinarily effective when the signal for the reference site and/or reference satellite is lost.

There is a need for a technique and related apparatus for more effectively utilizing the carrier phase measurements obtained in a surveying system utilizing signals broadcast from a plurality of orbiting satellites and detected by receivers located at two or more sites, one having known

position coordinates and the other having coordinates to be determined. The present invention fulfills this need.

#### Summary of the Invention

5 The present invention is embodied in an apparatus and related method for determining the position coordinates of a first predetermined site using signals broadcast from a plurality of orbiting satellites in a significantly more efficient and accurate manner than previously performed. A first receiver is located at the first predetermined site  
10 and a second receiver is located at a second, reference site having known position coordinates. The two receivers are adapted to receive the signals broadcast from the satellites and measure the carrier phase of each signal relative to their respective internal clocks, thus producing a plurality  
15 of measured carrier phase values.

These measured phase values are made at each of a succession of time points. To eliminate the effect of an unknown integral number of whole carrier cycles in each satellite/receiver link, the measured phase values are  
20 differenced over successive time points. This produces a plurality of measured phase difference values, which indicate the actual range changes that occur across successive time points. The apparatus further calculates the expected carrier phase of each satellite signal received  
25 by the respective first and second receivers, and differences these expected phase values over successive time points, to produce a plurality of expected phase difference values. Error values are then generated, equal to the difference between the expected phase difference values and  
30 the actual measured phase difference values for each of the successive time points.

Each of these error values has a certain sensitivity to incremental changes in the position coordinates of the associated site and in the internal clocks of the associated

receiver and satellite. The equations for the error values are expressed as functions of these latter parameters and incorporated into a predetermined matrix, at each successive time point. In accordance with the invention, the updating of the matrix at each time point includes a preliminary step of eliminating the dependence of matrix entries on previous values of the phase noise parameters for the receiver and satellite clocks. This elimination limits growth of the matrix, which otherwise would occur, and thus greatly improves the apparatus' efficiency. In the last step of the technique, the apparatus solves the matrix to determine the particular position coordinates for the first site that minimize the mean square value of the plurality of error values reflected in the matrix. This solution can be termed a hyperbolic solution, because it represents the best estimate of the single intersection point of a plurality of hyperbolic surfaces.

In one optional aspect of the invention, the matrix is formed using a triangular Householder algorithm. The matrix entries indicating sensitivity to the receiver and satellite clocks are located in a predetermined section of the matrix, such that when the matrix entries are updated, dependence on previously values of the phase noise parameters of the satellite and receiver clocks is eliminated simply by zeroing out the predetermined section. The matrix thereby remains the same size regardless of the number of times it is updated. This substantially simplifies the solving of the matrix to determine the first site's position coordinates.

The apparatus and procedure described above can represent merely the first stage in the determination of the first site's position coordinates. In an optional second stage, which is similar in many respects to the first stage, the process is modified to take into account the -0.5 correlation between the error values for successive time

points. This correlation arises because one of the terms in the respective equations defining the error value for successive time points is, by definition, the same. Thus, the second stage of the process appropriately whitens the successive error equations, making them substantially uncorrelated with each other, prior to incorporation into the matrix. Subsequently solving the matrix yields position coordinates that are even more accurate than the first stage solution. This second stage solution can be termed a Doppler pseudorange solution.

In an alternative embodiment of the invention, the second stage of processing can be used alone, without need for the first stage. Unless a fairly accurate estimation of the first site's position coordinates is initially made, however, this second stage will likely need to be iterated.

In an optional, final stage of the procedure, any ambiguity in the number of whole carrier cycles in the measured carrier phase signals is resolved. This third stage utilizes substantially the same algorithm as the second stage, described above. The only difference is the first measured phase signal is scaled by the square root of 2 before being incorporated into the matrix. The subsequent integrated Doppler measurements incorporated into the matrix are made to be uncorrelated with each other, i.e., whitened, as was done in the second stage, except that a cross-correlation coefficient of  $-.707$  is used, rather than  $-0.5$ . Solving the matrix after the information has been fully loaded into it provides a solution equivalent to one determined with knowledge of the actual number of whole carrier cycles in each link. Each satellite/receiver link is handled in a sequential fashion, with the correct resolution of each improving the probability of correctly resolving the next. The final solution can be termed a resolved lane solution.



Other aspects and advantages of the present invention will become apparent from the following description of the preferred embodiment, taken in conjunction with the accompanying drawings, which illustrate, by way of example, the principles of the invention.

#### Brief Description of the Drawings

Figure 1 is a schematic diagram (not to scale) of a surveying system having two receivers, one located at an unknown site and the other at a reference site having known coordinates, the receivers detecting signals broadcast from four orbiting satellites, to determine the coordinates of the unknown site;

Figure 2 is a simplified flowchart showing the operational steps performed by the apparatus of the invention in accurately determining the position coordinates of the unknown site of Figure 1;

Figure 3 is a schematic diagram of a square root information filter matrix used by the apparatus of the invention; and

Figures 4A and 4B are a flowchart depicting in more detail the operational steps performed in each of the first, second and third stages of the flowchart of Figure 2.

#### Description of the Preferred Embodiment

As shown in the accompanying drawings, this invention is embodied in an apparatus for accurately determining the position coordinates of a predetermined, unknown site 11 using signals broadcast from a plurality of orbiting satellites 13. The apparatus is particularly useful as part of the Global Positioning System (GPS). A first receiver 15 is located at the unknown site 11, and a second receiver 17 is located at a reference site 19 having known coordinates, which can be spaced many kilometers from the unknown site.

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Eight separate links 21 are thereby formed between the four satellites and two receivers.

5 The four satellites 13 all broadcast carrier signals having the same nominal frequency but each modulated by a unique pseudorandom digital code. The respective first and second receivers 15 and 17 include antennas 23 and 25 for receiving the superimposed, incoming modulated carrier signals and the receivers measure the phase angle of each. The phase angle measurements are transmitted on lines 27 and 29 from the  
10 respective receivers 15 and 17 to a data processor 31, for position determination.

The phase measurements are made periodically over an extended time (e.g., 30 minutes), and can be compacted in a conventional fashion. For example, the measurements can be  
15 made every 20 milliseconds and compacted into average values that are updated once per minute. Phase noise in the internal clocks of both the satellites and the receivers introduce uncertainties into these successive phase measurements.

20 In the past, the carrier phase measurements for each of the satellite/receiver links 21 were typically processed using a double differencing technique to detect the unknown site's position coordinates. This technique cancelled out the corrupting effects of phase noise in the internal clocks of  
25 the respective satellites and receivers and thereby led to a reasonably accurate position determination. For several reasons, however, this prior double differencing technique has not proven to be entirely satisfactory.

30 The apparatus of the invention avoids the drawbacks associated with the prior double differencing technique by expressly solving for the magnitude of the phase errors in the internal clocks of the satellites 13 and the receivers 15 and 17, rather than differencing those phase errors out.

This yields substantially improved results, in a highly efficient manner.

More particularly, the data processor 31 processes the successive sets of measured phase signals in a special three-stage process, which is depicted schematically in Figure 2. Three successive stages 33, 35 and 37 of the technique each yield progressively more accurate position determinations. The first stage yields what is called a hyperbolic solution, the second stage a Doppler pseudorange solution, and the third stage a resolved lane solution. These three stages are depicted in greater detail in Figures 4A and 4B. The three stages are substantially similar to each other, differing in only a few small, but significant ways, as will become apparent from the following description.

All three of the successive stages 33, 35 and 37 of the process expressly solve for the phase errors contributed by noise in the internal clocks of the satellites 13 and the receivers 15 and 17. Each stage determines the difference between the phase measurements of each satellite/receiver link 21 at successive time points, which corresponds to the range change between such time points. Each stage further computes the expected range change between the same time points, based on the best current estimate of the receiver site's coordinates and the known satellite orbits. The difference between corresponding ones of these phase difference measurements and phase difference estimates represents error values, whose equivalent equations are incorporated into a special square root information filter (SRIF) matrix. Each equation for each error value is given as follows:

$$\begin{aligned} \text{Error (E)} &= \text{Measured Range} - \text{Computed Range} \\ &= \text{Bias} + \frac{\partial E}{\partial X} \Delta X + \frac{\partial E}{\partial Y} \Delta Y + \frac{\partial E}{\partial Z} \Delta Z + \\ &\quad \frac{\partial E}{\partial \text{Clock}_{\text{sat}}} \Delta \text{Clock}_{\text{sat}} + \frac{\partial E}{\partial \text{Clock}_{\text{rcvr}}} \Delta \text{Clock}_{\text{rcvr}} \end{aligned}$$

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Where  $\begin{cases} X, Y + Z = \text{Site Coordinates} \\ \text{Clock} = \text{Satellite clock phase noise} \\ \text{Clock} = \text{Receiver clock phase noise} \end{cases}$

5 This equation is formed for all eight satellite/receiver links 21. The eight equations include 12 parameters, namely, the X, Y and Z coordinates for both receivers 15 and 17, the phase noise of the four satellite clocks, and the phase noise of the two receiver clocks. Because the reference receiver 17 has known coordinates and is defined  
10 to have a fixed reference clock, only eight of these parameters are unknown. They can be determined by solving the eight simultaneous equations. The bias terms in the eight equations are cancelled out by subtracting successive equations.

15 The SRIF matrix, which is depicted in simplified form in Figure 3, greatly simplifies the mathematical steps required to properly process the information. The lower-left triangular section 39 of the matrix is cleared to zero and remains in that state for the entire procedure. In each of  
20 the three stages of the Figure 2 process, the error equations for each of the successive time points are incorporated sequentially into the upper-right, triangular section 41 of the matrix. The particular entries that reflect sensitivity of the error values to phase noise in  
25 the internal clocks of the satellites 13 and the receivers 15 and 17 are intentionally located in the top rows of the matrix. The bottom rows reflect the sensitivity of the error values to the site coordinates. By so arranging the matrix entries, the dependence of the entries on previous  
30 values of the clock phase noise can be conveniently eliminated simply by zeroing out the pertinent top rows. Because of the nature of the SRIF matrix, this has no effect on the continued accuracy of the lower rows. In the last step in each stage of the procedure, the SRIF matrix is solved, to produce the particular corrections to the

previous estimate of the site coordinates that provide the minimum mean square error for the sum of the various error values.

5 In the first stage 33 of the procedure, the error equations are incorporated directly into the SRIF matrix. The resulting hyperbolic solution is a very accurate estimation of the known site's coordinates. In the second stage 35, the equations are whitened (i.e., made to be uncorrelated with each other) prior to incorporation into the matrix.  
10 This yields a slightly more accurate, Doppler pseudorange solution. Finally, in the third stage 37, the previous uncertainty in the number of whole carrier cycles  $n$  each satellite/receiver link 21 is expressly resolved, in a sequential fashion. The resulting final solution to the  
15 SRIF matrix is thereby an extremely accurate estimation of the unknown site's coordinates.

Figures 4A and 4B together are a simplified flowchart showing the operational steps performed by the data processor 31 in processing the successive carrier phase  
20 measurements derived for the various satellite/receiver links 21. The three stages 33, 35 and 37 of the overall process are depicted together in one figure so that the extensive similarities of the three stages can be readily determined. These similarities lead to significant savings  
25 in software development and in the sizing of the required processor.

Addressing first the steps performed in implementing the first stage 33 of the technique, it will be observed that a first step 43 of the procedure clears all of the locations  
30 in the SRIF matrix. Thereafter, at step 45, the matrix locations corresponding to the coordinates of the reference site 11 (Figure 1) are constrained to their known values by entering appropriate large values, e.g., one million. These matrix locations are located along the main diagonal of the

SRIF matrix, as indicated by the reference numeral 47 in Figure 3. The remaining matrix locations in the same rows retain their zero values.

5 In a following step 49 of the first stage 33, the program indexes to accept carrier phase measurements corresponding to the first of the plurality of time points. Subsequently in the program, when a loopback is made to this step 49, the program indexes to the next succeeding time point.

10 Next, in a step 51, the processor 31 clears the top rows of the SRIF matrix that carry information relating to the phase noise contributed by the various satellite and receiver clocks. In the initial loop through the program, these rows are already set to zero (by step 43). Also in step 51, the matrix location corresponding to phase noise contributed by  
15 the reference site clock is constrained by entering a large value (e.g., one million) into it. This location is the diagonal element identified by the reference numeral 53 in Figure 3.

The program next proceeds to a step 55, where it indexes to  
20 accept the particular carrier phase measurements corresponding to the first site 11. Later in the program, when looping back to this step, it indexes to the next succeeding site. In the preferred embodiment, just two sites are described, but it will be appreciated that the  
25 technique of the invention also has utility in processing the carrier phase measurements for a greater number of sites.

Next, at a step 57, the program indexes to accept carrier phase measurements corresponding to the first of the  
30 satellites 13. Later in the program, when looping back to this same step 57, the program indexes to the next succeeding satellite. In the preferred embodiment, four

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13

satellites are depicted, but it will be appreciated that any plurality will suffice.

At a succeeding step 59, the program computes the range from the currently-indexed satellite 13 to the currently-indexed receiver site 11 or 19, along with derivatives of the range equation with respect to the site's currently-estimated position coordinates. Initially, these estimated coordinates need not be accurate at all. In fact, they can even be initially set at the center of the earth. When the estimate is inaccurate to that degree, however, iteration of the first stage 33 of the technique will likely be required. Preferably, the initial estimate of the receiver's position is provided using the codes present on the incoming satellite signals. When this is done, iteration of the first stage is not required.

The derivatives computed in the steps 59 are used as the coefficients in a conventional Taylor series expansion of the range equation. First order terms are all that are ordinarily required, so that the range equation is a mere linear function of the estimated site coordinates.

In a following step 61, it is determined whether or not the carrier phase measurements currently being processed correspond to the first time point. If they do, the program proceeds to a step 63, where it is determined whether or not the measurements for all of the satellites 13 have been processed for the currently-indexed site. If they have not been, the program loops back to the step 57, where it indexes to the next succeeding satellite. If, on the other hand, the measurements for all of the satellites have been processed, the program advances to a step 65, where it is determined whether or not the measurements for all of the sites have processed. If they have not, the program loops back to the step 55, where it indexes to the next succeeding site. If, on the other hand, the measurements for all of

the sites have been processed, the program loops back to the step 49, where it indexes to the next succeeding (i.e., the second) time point.

5 With reference again to step 61, if it is determined that the phase measurements currently being processed are for the second and subsequent time points, the processor 31, at step 67, computes the differences since the previous time points of the phase measurements, computed ranges, and derivatives, for the satellite/receiver site currently being processed.  
10 These differences are used to form the error term and measurement equation.

In a following step 69, a triangular Householder algorithm is used to incorporate the measurement equation into the SRIF matrix. This is a conventional algorithm that is  
15 described in detail in a book by Gerald J. Bierman, entitled "Factorization Method for Discrete Sequential Estimation", Academic Press, 1977. The algorithm Bierman describes is preferably modified to permit operation on a single measurement equation at a time, rather than all  
20 simultaneously.

After the SRIF matrix has been incorporated with the measurement equation in step 69, the program advances to step 71, where it is determined whether or not the measured phase signals for all of the satellites 13 (for the current  
25 site 11 or 19 and current time point) have been processed. If not, a return is made to the step 57, where the program indexes to the next succeeding satellite. If, on the other hand, the measured phase signals for all of the satellites have been processed, the program advances to a step 73,  
30 where it is determined whether or not the signals corresponding to all of the receiver sites (for the current time point) have been processed. If not, a return is made to step 55, where the program indexes to the next succeeding site.

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If, on the other hand, it is determined at step 73 that the signals corresponding to all of the receiver sites have been processed, the SRIF matrix is solved at step 75, to determine the phase errors of the various satellite and receiver clocks. The measured phase signals for the current time point are then all corrected to reflect these clock phase errors.

Thereafter, at step 77, the program determines whether or not the measured phase signals for all of the successive time points have been processed. If not, a return is made to step 49, where the program indexes to the next succeeding time point.

If, on the other hand, it is determined at step 77 that the measured phase signals for all of the successive time points have been processed, the program advances to a step 79, where the SRIF matrix is solved to determine the appropriate corrections to the site coordinates. These corrections are then made to the earlier best estimate of these coordinates, and the first stage of the procedure has been completed. The resulting corrected position coordinates for each unknown site, which, in the preferred embodiment, is merely the single site 11, in fact corresponds to the particular coordinates that represent the least mean square error in the plurality of error signals computed for the various measurements. These coordinates represent the particular location that is the best (i.e., least mean square error) estimate of the intersection between the various hyperboloids defined by the range difference equations.

The corrected position coordinates for the unknown site 11 provided by the first stage 33 of the procedure are sufficiently accurate for many practical applications. Accuracy to within about 5-10 cm can be expected from data accumulated over an approximately 30 minute span. One factor contributing to the limited accuracy is that the

successive time difference or Doppler range equations are not entirely uncorrelated with each other. This is because each successive difference measurement has, as one of its terms, the range equation for the same satellite, receiver and time point. The correlation is -0.5. The corrections to the position coordinates determined by solving the SRIF matrix could be improved if this correlation were eliminated. This is what is accomplished in the second stage 35 of the procedure.

As previously mentioned, the second stage 35, which yields the Doppler pseudorange solution, is very much similar to the first stage 33, which yields the hyperbolic solution. With reference to Figures 4A and 4B, it will be observed that the second stage differs from the first stage in only two minor, but significant, respects. First, an extra step 81 is included immediately after the step 45 of constraining the coordinates of the reference site. In particular, this extra step clears the correlation coefficient for each satellite/site pair. This is done so that the successive difference measurements can be appropriately adjusted to make them substantially uncorrelated with each other.

The second difference between the second stage 35 and the first stage 33 is the inclusion of two additional steps 83 and 85 immediately after the formation at step 67 of the error term and measurement equation. In particular, step 83 incorporates the clock phase noise portion of the current measurement into the SRIF matrix. Step 85 then whitens the measurement by computing the suitable measurement weighting factor from the correlation coefficient and adjusting the measurement by combining it with the previous measurement multiplied by the correlation coefficient. The whitening procedure is described in the Bierman reference identified above. The measurement is also scaled by the weighting factor. The adjusted and scaled measurement is saved for use at the next time point. After step 85, stage two 35 of

the procedure continues with step 69 and subsequent steps, in the same manner as was described above with respect to stage one 33.

5 Solving the SRIF matrix at step 79 of stage two 35 provides corrections to the position coordinates of the unknown site 11 to make the coordinate estimate even more accurate than after stage one 33. Again, this increased accuracy results from taking into account the -0.5 correlation between the successive signal differences for each satellite/receiver  
10 link 21.

Although the corrected site coordinates resulting from the data processor's implementation of stage two 35 of the procedure are sufficiently accurate for many practical applications, the accuracy of these coordinates can be  
15 enhanced even further. This enhancement can be achieved by taking into account the whole cycle bias in each of the successive measured phase signals for all of the satellite/receiver links 21. It will be recalled that these biases were automatically cancelled out in both stage one 33  
20 and stage two 35 by subtracting the error signals for successive time points for each link. Although the biases were cancelled out, this particular technique for doing so led to minor inaccuracies in the resulting coordinate corrections. Stage three 37 of the procedure specifically  
25 solves for the bias in each link.

With reference again to Figures 4A and 4B, it will be observed that stage three 37 of the procedure is very much similar to stages one and two, described above. These differences are described below.

30 First, stage three 37 includes an additional step 87 that immediately follows the stage two step 81 of clearing the correlation coefficient for each satellite/receiver pair. In particular, step 87 provides an additional indexing

through the plurality of satellites. The first time the step is implemented, the program indexes to the first satellite. Thereafter, when looping back to this step, the program indexes to the next succeeding satellite. Following  
5 step 87, the program continues with steps 49-61, as described above with respect to stage one 33.

In a further difference between the third stage 37 and the earlier-described second stage 35, additional steps 89 and 91 are included following a no answer in step 61, i.e., a  
10 determination that the processor 31 is not currently processing signals for the first time point. At step 89, it is determined whether or not the satellite number in the extra loop (selected in step 87) is less than the satellite number selected in the main loop (in step 57). If it is, a  
15 return is made to step 49, where an index is made to the next time point. On the other hand, if it is determined that the extra loop is not less than the first loop, the program proceeds to step 91, where the error term is computed by keeping only the fractional portion of the  
20 difference between the measured value signal and the computed range. The measurement equation is then formed and the correlation coefficient set to -0.707. Thereafter, the program proceeds to steps 83 and 85, which is the same as in the second stage 35 of the procedure.

25 The program then continues in the same fashion as in the second stage 35, resulting ultimately in the solving of the SRIF matrix at step 79. In this third stage 37, however, this last step 79 is followed by a step 93, in which it is determined whether or not the extra loop through the  
30 plurality of satellites 31 has been completed. If not, the program loops back to the step 87, in which the satellite number in the extra loop is indexed by one.

When it is determined at step 93 that all of the satellite numbers have been indexed through in this extra satellite

loop, the entire procedure is completed. The last corrections made to the position coordinates of the unknown site are, at this time, in their most accurate state.

5 The third stage 37 or resolved lane solution is thus divided into as many sub-solutions as there are satellites 31. The result of each sub-solution is an improved solution that has a higher probability of correctly resolving the whole cycle ambiguity of the next satellite to be incorporated into the solution. The final solution is attained when all of the  
10 satellites have been incorporated into the solution and the whole cycle ambiguity values resolved.

Attached as an Appendix is a program listing, written in Fortran, of one suitable computer program useful in implementing the present invention.

15 In an alternative embodiment of the invention, the SRIF matrix is replaced by a  $A^T A$  matrix, where A is a standard matrix comprising the coefficients of the various error equations. This embodiment is less desirable, however, because it requires a relatively complicated Gauss  
20 elimination procedure to eliminate its dependence on prior clock phase noise parameters.

It should be appreciated from the foregoing description that the present invention provides a significantly improved technique for determining the position coordinates of an  
25 unknown site using signals broadcast from a plurality of orbiting satellites and using the known position coordinates of a predetermined reference site. The technique improves upon prior techniques by expressly solving for the phase noise in the internal clocks of the satellites and  
30 receivers. The procedure can be broken down into three distinct stages, each differing in only a small, but significant, way from the preceding stage. Each successive

stage yields a slightly more accurate estimation of the unknown site's position coordinates.

5 Although the invention has been described in detail with reference to the presently preferred embodiment, those of ordinary skill in the art will appreciate that various modifications can be made without departing from the invention. Accordingly, the invention is defined only by the following claims.

## I CLAIM:

- 1 1. A method for determining the position coordinates of a  
2 first receiver using signals broadcast from a plurality  
3 of orbiting satellites, wherein a second receiver is  
4 located at a reference location having known position  
5 coordinates, and wherein the first and second receivers  
6 each receive the plurality of broadcast signals and  
7 detect the carrier phase of each signal relative to an  
8 internal clock, the method comprising steps of:  
  
9 measuring the phase of the carrier signal received by  
10 each of the first and second receivers from each  
11 satellite, to produce a plurality of carrier phase  
12 measurements;  
  
13 calculating the expected phase of the carrier signal  
14 received by each of the first and second receivers from  
15 each satellite, based on the satellite's orbit and an  
16 estimate of the receiver's position, to produce a  
17 plurality of carrier phase estimates;  
  
18 repeating the steps of measuring and calculating at  
19 regular time intervals, to produce a plurality of  
20 carrier phase measurements and a plurality of carrier  
21 phase estimates at each of a succession of time points;  
  
22 determining the difference between the changes in  
23 carrier phase measurements and the changes in carrier  
24 phase estimates over successive time points for each  
25 satellite/receiver pair, to produce a plurality of  
26 error values for each time point;  
  
27 defining each of the plurality of error values for each  
28 time point to be equal to predetermined function of the  
29 signal's sensitivity to small changes in the position

30 coordinates of the corresponding receiver and in the  
31 clocks of the corresponding receiver and satellite;

32 forming and updating at each time point a predetermined  
33 matrix based on the plurality of error equations for  
34 each time point;

35 the step of forming and updating including a step of  
36 eliminating dependence on previous values of the  
37 receiver and satellite clocks; and

38 solving the matrix to determine the particular position  
39 coordinates for the first receiver that minimize the  
40 mean square value of the plurality of error values.

1 2. A method as defined in claim 1, wherein:

2 the step of forming includes a step of forming a square  
3 root information filter matrix using a triangular  
4 Householder algorithm;

5 the matrix entries indicating sensitivity to the  
6 receiver and satellite clocks are located in a  
7 predetermined section of the matrix; and

8 the step of updating includes a preliminary step of  
9 zeroing the predetermined section of the matrix.

1 3. A method as defined in claim 1, wherein the step of  
2 forming includes a step of modifying the successive  
3 sets of error equations to make the sets substantially  
4 uncorrelated with each other.

1 4. A method as defined in claim 3, wherein the step of  
2 modifying includes a step of whitening the successive  
3 sets of error equations.



1 5. A method as defined in claim 1 and further including  
2 steps following the step of solving, of:

3 modifying the plurality of error equations to make the  
4 successive equations for each satellite/receiver pair  
5 substantially uncorrelated with each other, thereby  
6 producing a plurality of modified error equations for  
7 each time point;

8 forming and updating the predetermined matrix based on  
9 the plurality of modified error equations for each time  
10 point; and

11 solving the matrix to determine the particular position  
12 coordinates for the first receiver that minimize the  
13 mean square value of the plurality of error values.

1 6. A method as defined in claim 5, wherein the step of  
2 modifying includes a step of whitening the successive  
3 sets of error equations.

1 7. A method as defined in claim 5, and further including  
2 steps, following the second step of solving, of:

3 resolving any ambiguity in the integral number of  
4 carrier cycles between each satellite and receiver  
5 pair; and

6 determining the particular position coordinates for the  
7 first receiver that minimize the mean square value of  
8 the plurality of error values, taking into account the  
9 integral number of carrier cycles determined, in the  
10 step of resolving, to be between each satellite and  
11 receiver pair.

- 1 8. A method as defined in claim 4, wherein the matrix used  
2 in the step of forming remains the same size following  
3 each updating.
- 1 9. A method as defined in claim 1, wherein the matrix used  
2 in the step of forming remains the same size following  
3 each updating.
- 1 10. Apparatus for determining the position coordinates of a  
2 first predetermined site using signals broadcast from a  
3 plurality of orbiting satellites, the apparatus  
4 comprising:
- 5 a first receiver located at the first site;
- 6 a second receiver located at a second, reference site  
7 having known position coordinates;
- 8 wherein the first and second receivers both have  
9 internal clocks and are adapted to receive the signals  
10 broadcast from the plurality of satellites and measure  
11 the carrier phase of each signal relative to their  
12 respective internal clocks, to produce a plurality of  
13 carrier phase measurements for each of a succession of  
14 time points;
- 15 means for calculating the expected phase of each  
16 carrier signal received by the first and second  
17 receivers from each satellite, based on the satellite's  
18 orbit and an estimate of the receiver's position, to  
19 produce a plurality of carrier phase estimates for each  
20 of the succession of time points;
- 21 matrix means for forming a predetermined matrix that  
22 defines each of the plurality of error values to be  
23 equal to a predetermined function of its own  
24 sensitivity to small changes in the position

25 coordinates of the corresponding site and in the  
26 internal clocks of the corresponding receiver and  
27 satellite;

28 means for determining changes in the carrier phase  
29 measurements and changes in the carrier phase estimates  
30 over successive time points, and for determining the  
31 differences between corresponding pairs of such  
32 measurements and estimates for each satellite/receiver  
33 pair and for each time point, to produce a plurality of  
34 error values for each time point;

35 wherein the matrix means includes means for updating  
36 the matrix to reflect the plurality of error equations  
37 for each time point, the updating means including means  
38 for eliminating dependence on previous values of the  
39 receiver and satellite clocks; and

40 means for solving the matrix to determine the  
41 particular position coordinates for the first site that  
42 minimize the mean square value of the plurality of  
43 error values.

1 11. Apparatus as defined in claim 10, wherein:

2 the predetermined matrix formed by the matrix means is  
3 a square root information filter matrix;

4 the matrix means includes means implementing a  
5 triangular Householder algorithm to incorporate the  
6 successive error equations into the matrix;

7 the matrix entries indicating sensitivity to the  
8 receiver and satellite clocks are located in a  
9 predetermined section of the matrix; and

10 the matrix means further includes means for zeroing the  
11 predetermined section of the matrix.

1 12. Apparatus defined in claim 10, wherein the matrix means  
2 further includes means for modifying the successive  
3 sets of error equations to make the sets substantially  
4 uncorrelated with each other.

1 13. Apparatus as defined in claim 12, wherein the means for  
2 modifying includes means for whitening the successive  
3 sets of error equations.

1 14. Apparatus as defined in claim 10, and further  
2 including:

3 means for modifying the plurality of error equations to  
4 make the successive equations for each  
5 satellite/receiver pair substantially uncorrelated with  
6 each other; thereby producing a plurality of modified  
7 error equations for each time point;

8 means, operable after the first means for solving, for  
9 forming and updating a second predetermined matrix  
10 based on the plurality of modified error equations for  
11 each time point; and

12 means for solving the second matrix to determine the  
13 particular position coordinates for the first receiver  
14 that minimize the mean square value of the plurality of  
15 error values.

1 15. Apparatus as defined in claim 14, wherein the means for  
2 modifying includes means for whitening the successive  
3 sets of error equations.

1 16. Apparatus as defined in claim 14, and further  
2 including:

3 means, operable after the second means for solving, for  
4 resolving any ambiguity in the integral number of  
5 carrier cycles between each satellite and receiver  
6 pair; and

7 means for determining the particular position  
8 coordinates for the first receiver that minimize the  
9 mean square value of the plurality of error values,  
10 taking into account the integral number of carrier  
11 cycles, determined by the means for resolving, to be  
12 between each satellite and receiver pair.

1 17. Apparatus as defined in claim 14, wherein the first and  
2 second predetermined matrices remain the same size  
3 following each updating.

1 18. Apparatus as defined in claim 10, wherein the  
2 predetermined matrix remains the same size following  
3 each updating.

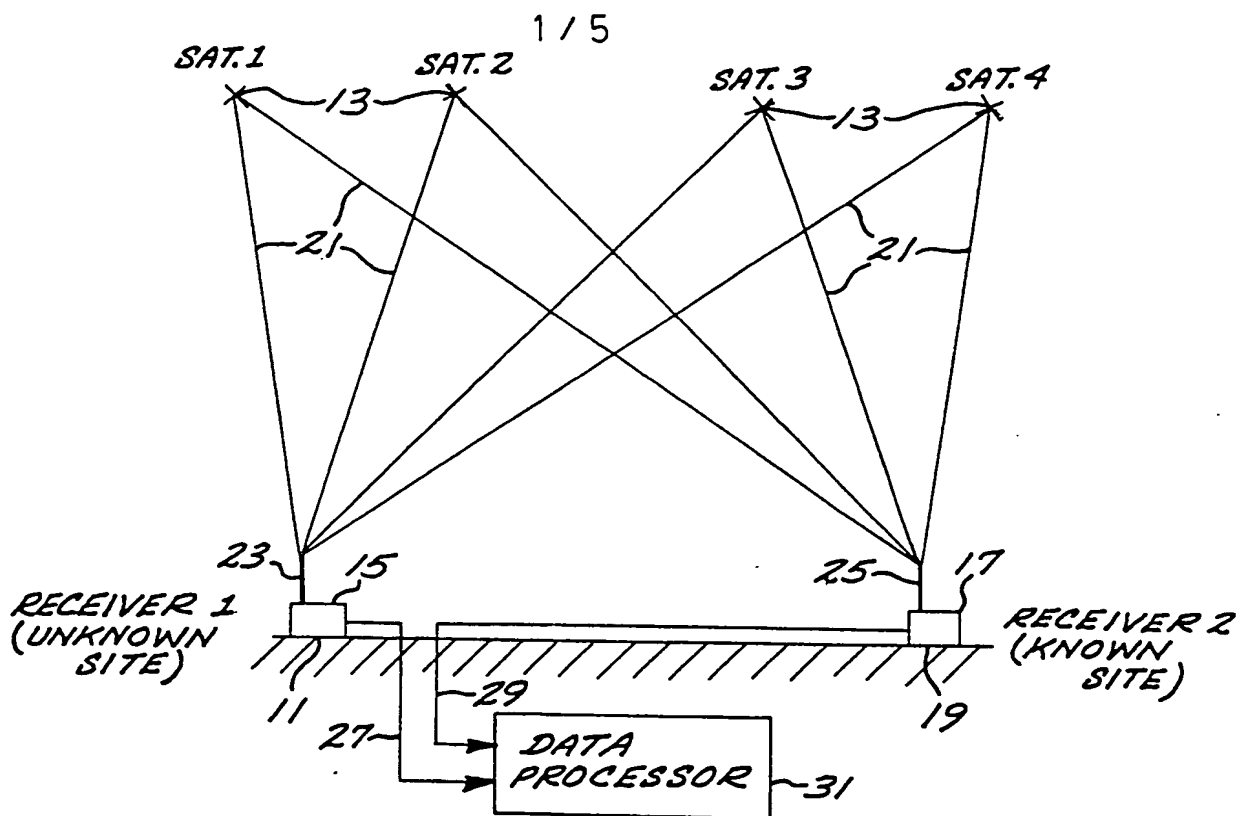


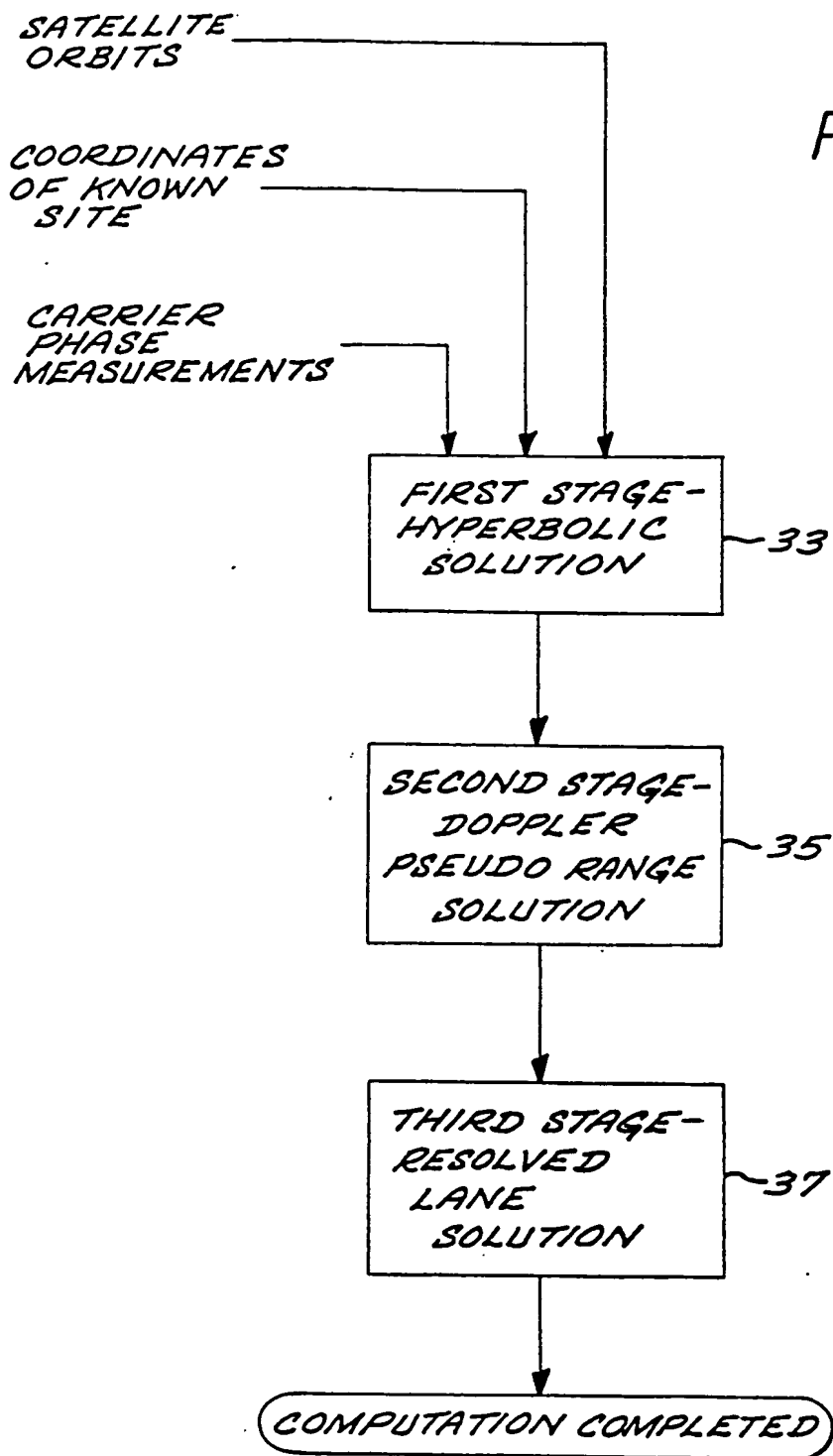
FIG. 1

FIG. 3

[illegible]

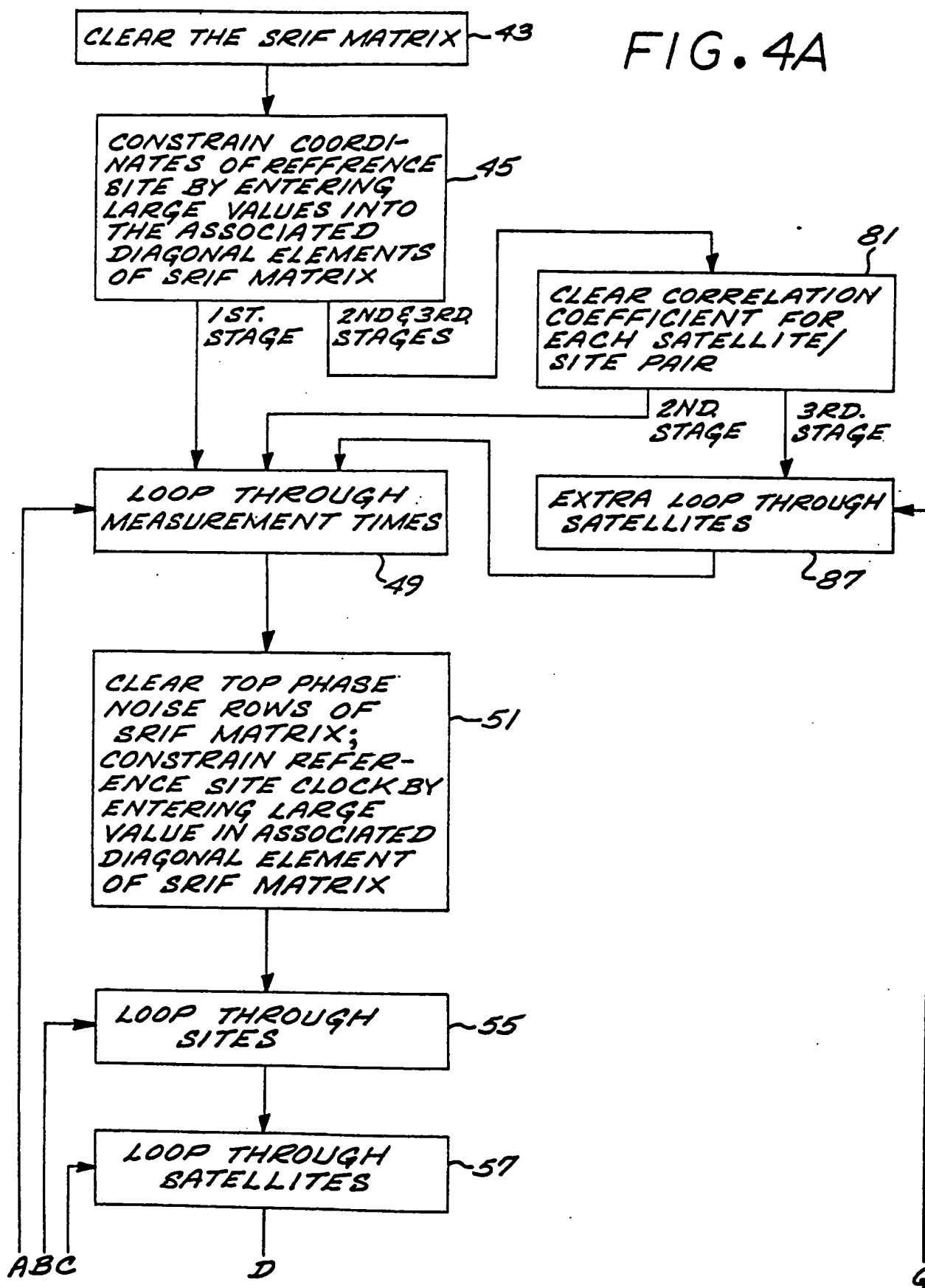
2 / 5

FIG. 2



3 / 5

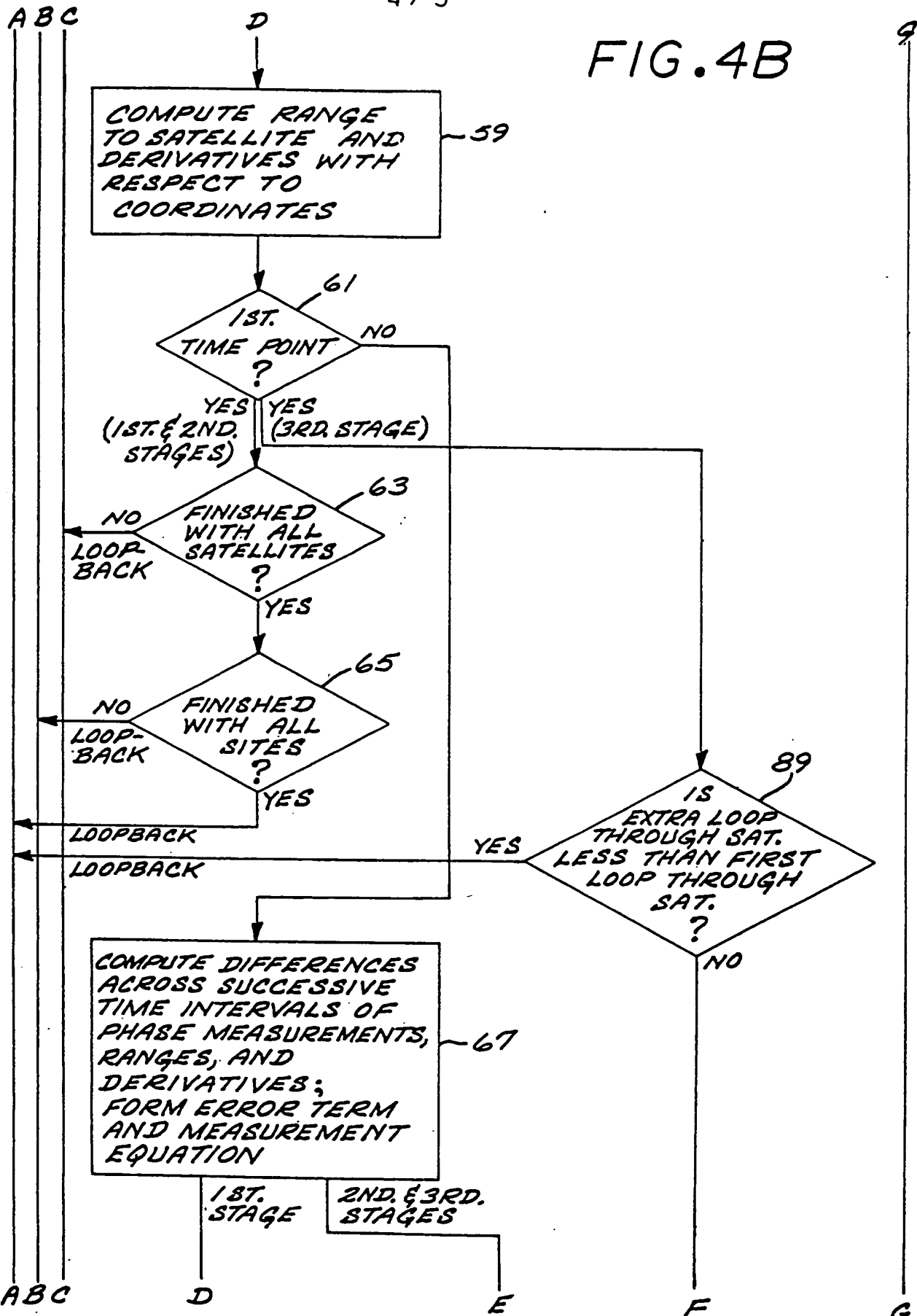
FIG. 4A



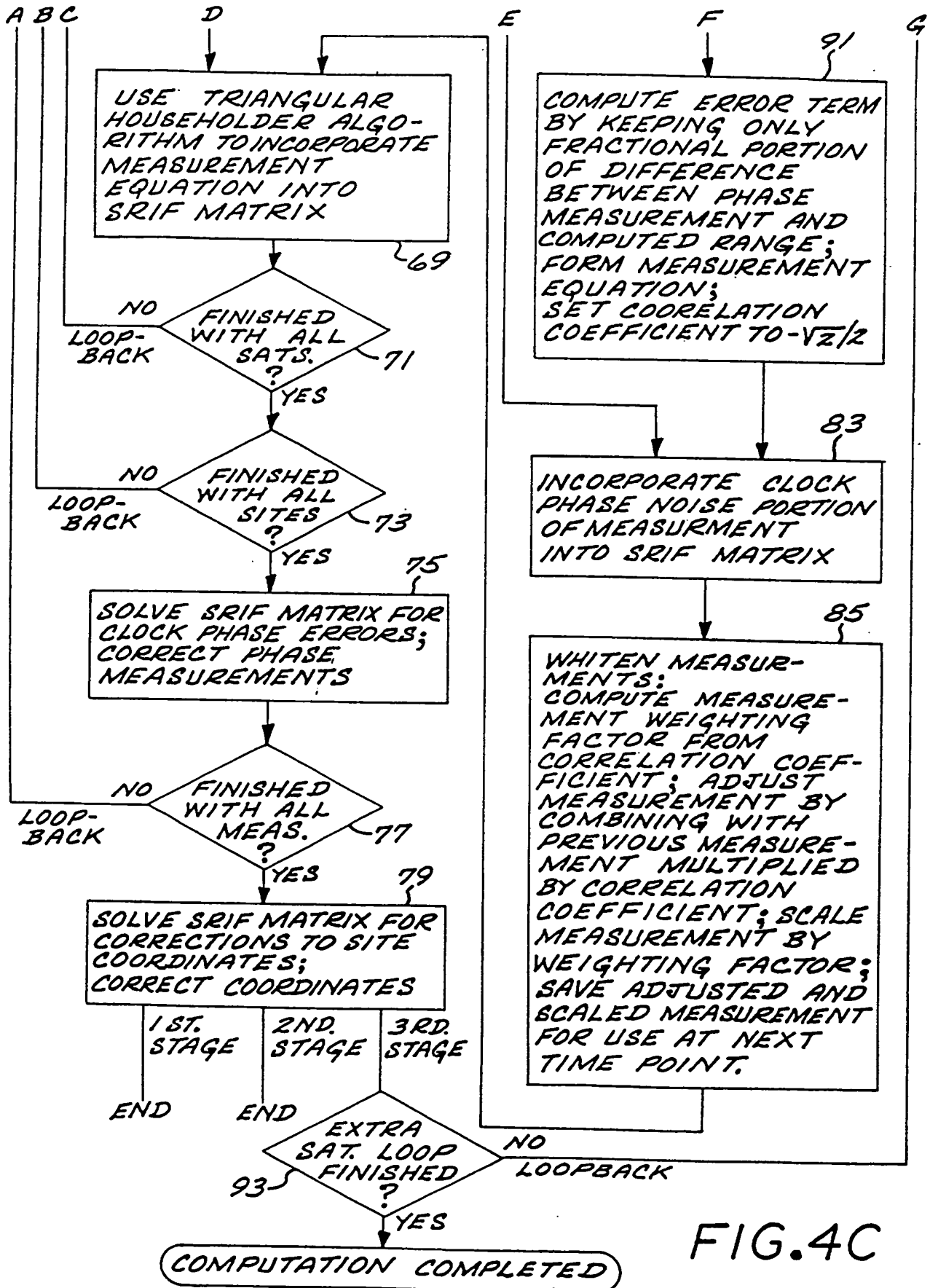


4 / 5

FIG. 4B



5 / 5



# INTERNATIONAL SEARCH REPORT

International Application No **PCT/US87/00825**

<b>I. CLASSIFICATION OF SUBJECT MATTER</b> (If several classification symbols apply, indicate all) <sup>3</sup> According to International Patent Classification (IPC) or to both National Classification and IPC IPC 4: <b>H04B 7/19</b> U.S. Cl. : <b>342/356</b>		
<b>II. FIELDS SEARCHED</b>		
Classification System	Minimum Documentation Searched <sup>4</sup>	
U.S.,	Classification Symbols <b>342/356, 357, 394, 396; 364/452</b>	
Documentation Searched other than Minimum Documentation to the Extent that such Documents are Included in the Fields Searched <sup>5</sup>		
<b>III. DOCUMENTS CONSIDERED TO BE RELEVANT</b> <sup>14</sup>		
Category <sup>6</sup>	Citation of Document, <sup>15</sup> with indication, where appropriate, of the relevant passages <sup>17</sup>	Relevant to Claim No. <sup>18</sup>
X A A	US, A, 3906204 (Rigdon et al.) 16 September 1975 US, A, 4150335 (Cooperman) 17 April 1979 US, A, 4315262 (Acampora et al.) 9 February 1982	1-18 1-18 1-18
<div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <p><sup>9</sup> Special categories of cited documents: <sup>15</sup></p> <p>"A" document defining the general state of the art which is not considered to be of particular relevance</p> <p>"E" earlier document but published on or after the international filing date</p> <p>"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)</p> <p>"O" document referring to an oral disclosure, use, exhibition or other means</p> <p>"P" document published prior to the international filing date but later than the priority date claimed</p> </div> <div style="width: 45%;"> <p>"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention</p> <p>"X" document of particular relevance: the claimed invention cannot be considered novel or cannot be considered to involve an inventive step</p> <p>"Y" document of particular relevance: the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art.</p> <p>"Δ" document member of the same patent family</p> </div> </div>		
<b>IV. CERTIFICATION</b>		
Date of the Actual Completion of the International Search <sup>2</sup>	Date of Mailing of this International Search Report <sup>2</sup>	
<b>19 May 1987</b>	<b>15 JUN 1987</b>	
International Searching Authority <sup>1</sup>	Signature of Authorized Officer <sup>20</sup>	
<b>TSA/US</b>	<i>D. E. Hayes</i> <b>D. E. Hayes</b>	